How Supercritical String Cosmology (SSC) affects LHC

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OUTLINE

● WHY NON-EQUILIBRIUM STRINGY COSMOLOGIES? Theoretical models and ideas on the Early Universe: Collision of Brane worlds as cosmically catastrophic reason for non-criticality


● WHAT KIND OF PHENOMENOLOGY?
  (I) Astro-Particle Phenomenology This talk
  Off-shell (non-critical string) and dilaton corrections to Boltzmann Equation for relic abundances, Modifications on constraints for Cosmologically appealing particle physics models (e.g. supersymmetric)
  (II) Astrophysics Tests:
    (i) Supernovae Data This talk
    (ii) Cosmic Microwave Background Anisotropies (CMB) (WMAP etc.)
    (iii) Baryonic Acoustic Oscillations .......
Evidence for Dark Energy

Recent Astrophysical Evidence for Dark Energy (acceleration of the Universe (SnIA), CMB anisotropies (WMAP...))
PHYSICS: WMAP and Dark Matter

WMAP results so far:

- Disfavor strongly hot dark matter (neutrinos), $\Omega_{\nu} h^2 < 0.0076$ ($< m_{\nu} > < 0.23$ eV).
- Warm Dark Matter (gravitino) disfavoured by evidence for re-ionization at redshift $z \sim 20$.
- Cold Dark Matter (CDM) remains: axions, supersymmetric dark matter (lightest SUSY particle (LSP)), superheavy (masses $\sim 10^{14\pm6}$ GeV)

WMAP results: $\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$ (matter), $\Omega_b h^2 = 0.0224 \pm 0.0009$ (baryons), hence, assuming CDM is the difference, $\Omega_{CDM} h^2 = 0.1126^{+0.0161}_{-0.0181}$, (2\sigma level).
WHY NON-EQUILIBRIUM STRING THEORY

Several reasons:

- **String theory is a mathematically consistent model of Quantum Gravity.**

- **Quantum Gravity is ultimately linked with Cosmology.**

- **Cosmically-Catastrophic initial events, such as Big-Bang, or Collision of Brane Worlds (modern version of strings), may induce Non-Equilibrium Physics $\implies$ Relaxation Models for Dark Energy:**
  present dark energy remnant of initial non-equilibrium catastrophic event

- **Formal description:** Non-Critical (non-conformal, Liouville) Strings describing excitations on our brane world: Non-critical strings approach asymptotically (in time) critical (conformal) strings (equilibrium points), Asymptotically vanishing dark energy, perturbative $S$-matrix (corner stone of critical strings) well-defined...

- **Consequences:**
  
  (i) of Stringy description: Dilaton couplings with matter, time-dependent dilaton source terms in Boltzmann equation for relic abundances

  (ii) of Non-critical stringy description: **Off-shell Einstein Cosmological equations (relaxation scenario), relaxing to zero dilaton dark energy, current acceleration, extra source terms in Boltzmann equation $\implies$ further modifications on relic abundances.**
Origins of String Cosmology, about 20 years ago:

Discovery of the first exact cosmological solution of (supercritical) strings:

linear (in time) dilaton background \[ \phi \propto -QX^0 \]
that leads in the Einstein frame to a linear expanding universe

\[ \alpha(t) \propto t \]

By I. Antoniadis, K. Bachas, J. Ellis, D.V. Nanopoulos (ABEN):


Liouville-String (Dissipative, Non-Equilibrium) Cosmology: Formalism

Space of background space-time fields, over which strings propagate:

\( \{ g^i \} = \{ \text{graviton} = G_{\mu \nu}, \text{matter}, \ldots \} \), and Dilaton \( \phi(X^0, \rho) \) \( \rho \) = Liouville mode, Irreversible Relaxation Flow (RG) between string vacua (equilibrium points) (Irreversibility \( \leftrightarrow \) Zamolodchikov C-theorem).

\( \rho \) is ESSENTIAL in restoring conformal invariance, perturbed by a NON-EQUILIBRIUM PROCESS, e.g. Catastrophic Cosmic Events (Brane World Collision), or space-time foam (microscopic black holes, space-time defects etc.), ...
Stringy $\sigma$-model **GENERALIZED CONFORMAL INVARiance**:

$$\ddot{g}^i + Q \dot{g}^i = -\beta^i$$

$\dot{g}^i = \frac{dg^i}{d\rho_0}$, $\rho_0$ world-sheet zero mode of the Liouville field. NB: “Liouville friction term”

$\propto Q =$ central charge deficit, $\beta^i =$ $\sigma$-model Renormalization Group (RG) $\beta$-functions,

e.g. for gravitons, to $O(\alpha')$ ($\alpha' =$ Regge slope): $\beta^G_{\mu\nu} = R_{\mu\nu}$ (Ricci tensor)

**CONFORMAL INVARiance CONDITIONS** $\beta^i = 0 \iff$ EQUATIONS OF MOTION FROM ON-SHELL TARGET-SPACE ACTION

$$-\beta^i = -\delta S/\delta g^i = 0$$

**GENERALIZED (LIOUVILLE) CONFORMAL INVARiance CONDITIONS** $\iff$

EQUATIONS OF MOTION FROM OFF-SHELL TARGET-SPACE ACTION

$$\beta^i = \delta S/\delta g^i = \ddot{g}^i + Q \dot{g}^i \neq 0$$

Energetics (some supercritical models): Time-like Liouville $\rho \iff$ (Cosmic)Time $X^0(t)$

RG irreversibility = Fundamental time arrow, CPT Violation (Ellis, NM, Nanopoulos)
A Specific Model

Colliding Brane Worlds with Recoil (Ellis, NM, Nanopoulos, Westmuckett)

Relative recoil velocity $V$ of branes
Adiabatic collisions allow for stringy computation of brane potential to leading order in $V$

Identical branes (same tension) potential is

$$V_{\text{potential}} \propto V^4$$

Stringy Excitations on brane world
Feel central charge deficit

$$Q_{\text{initial}}^2 = Q(V^4)$$

RELAXATION PROCESS AFTER COLLISION

$$Q(t) \sim V^2$$

EFFECTIVE LIOUVILLE STRINGY COSMOLOGY ON THE BRANE

$\phi$, $G_{\mu\nu}$, $Q(t)$

Logarithmic (super)Conformal Field Theory Techniques to compute scaling with cosmic time, PLUS identification of Liouville mode (IRREVERSIBLE local RG scale on world-sheet) with time (Ellis, Mavromatos, Nanopoulos)

Collision $\leftrightarrow$ Non-Equilibrium, Irreversible
Dissipative Liouville Q-Cosmologies

Robertson-Walker Space times. Effective 4-d action with matter $I_m$ and radiation in Einstein frame

$$S^{(4)} = \frac{1}{2\alpha} \int d^4x \sqrt{-G} \left[ e^{-\Psi(\phi)} R(G) + Z(\phi)(\nabla \phi)^2 + 2\alpha' \phi(\phi) \right] - \frac{1}{16\pi} \int d^4x \sqrt{G} \frac{1}{\alpha(\phi)} F_{\mu\nu}^2 - I_m(\phi, G, \text{matter})$$

including string loops, $e^{\Psi(\phi)} = c_0 e^{-2\phi} + c_1 + c_2 e^{2\phi} + \ldots$, $Z(\phi) = 4 + \ldots$, ...

$V(\phi) = 2Q^2 e^{2\phi} + \tilde{V}$, $\tilde{V} = \alpha_3 e^{3\phi} + \beta_4 e^{4\phi} * \ldots$

Off-Shell Liouville Equations: $(\rho_\phi = (\dot{\phi})^2 + V(\phi)/2$, $p_\phi = (\dot{\phi})^2 - V(\phi)/2$)

$$3H^2 = \rho_m + \rho_\phi + \frac{e^{2\phi}}{2} J_\phi,$$

$$2 \frac{dH}{dt} = -\rho_m - 2\dot{\phi} + \rho_m - p_\phi + a^{-2}(t) J_{ii}, \quad i = 1, 2, 3,$$

$$\frac{d^2\phi}{dt_E^2} + 3H \frac{d\phi}{dt_E} + \frac{1}{4} \frac{\delta V}{\delta \phi} + \frac{1}{2} (\rho_m - 3p_m) = -\frac{3}{2} J_{ii} - \frac{e^{2\phi}}{2} J_\phi,$$

$$J_\phi = e^{-2\phi}(\dot{\phi} - \dot{\phi}^2 + Qe^{\phi} \ddot{\phi})$$, $J_{ii} = 2a^2 \left( \dot{\phi} + 3H\dot{\phi} + \dot{\phi}^2 + (1 - q)H^2 + Qe^{\phi}(\dot{\phi} + H) \right)$.

Matter (non) Conservation equations:

$$\dot{\rho}_m + 3H(\rho_m + p_m) + \dot{\phi}(\theta V(\phi))/2\theta Q - \phi(\rho_m - 3p_m) = 6(H + \dot{\phi})a^{-2} J_{ii}$$
EVOLUTION OF A LIOUVILLE UNIVERSE

In Liouville string (Non-Equilibrium, off-shell) Dark Energy Models, Dilaton Dark Energy may be negligible at NUCLEOSYNTHESIS epoch.

Conformal Field Theory (Logarithmic CFT, in brane recoil models) → asymptotic scaling with cosmic time $\sim 1/t^2$  
J. Lopez and D.V. Nanopoulos, ’94–’95, Ellis, Mavromatos, Nanopoulos ’95

NB: Cosmic Time $\leftrightarrow$ world-sheet

Renormalization Group (RG) local Scale (Liouville mode), Irreversible (Zamolodchikov C-theorem)!
Numerical Treatment of Modified Cosmological Equations (Diamandis, Georgalas, Lahanas, NM, Nanopoulos, hep-th/0605181)

**Left panel:** The dilaton $\phi$, the (square root of the) central charge deficit $Q$ and the ratio $a/a_0$ of the cosmic scale factor as functions of the Einstein time $t_{Einstein}$. The present time is located where $a/a_0 = 1$ and in the figure shown corresponds to $t_{today} \simeq 1.07$. The input values for the densities are $\rho_b = 0.238$, $\rho_c = 0.0$ and $w_c$ is 0.5. The dilaton value today is taken $\phi = 0.0$. **Right panel:** The values of $\Omega_i \equiv \rho_i/\rho_c$ for the various species as functions of $t_{Einstein}$.
Q-Cosmology Predictions on Cosmological Parameters

Left panel: Ratios of $\Omega$'s for the dilaton ($\phi$), exotic matter ($e$) and the non-critical terms ("noncrit") to the sum of "dust" and radiation $\Omega_d + \Omega_r$ densities. Right panel: The quantities $\rho_b \, a^3$, for "dust", $\rho_r \, a^4$ and $\rho_e \, a^2$ as functions of $t_{Einstein}$. 
Q-Cosmology Predictions on Cosmological Parameters

Left panel: The deceleration $q$ and the dimensionless Hubble expansion rate $\dot{H} = \frac{\dot{H}}{\sqrt{H_0^2}}$ as functions of $t_{\text{Hubble}}$. Right panel: The derivative of the dilaton and its ratio to the dimensionless expansion rate.
The deceleration as function of redshift values in the range $z = 0.0 - 1.0$. 
Q-Cosmology Predictions on Cosmological Parameters

**Approximate analytical treatment at late eras ( z < 2):** (Ellis, NM, Mitsou, Nanopoulos, astro-ph/0604272)

Central charge deficit (off-equilibrium, non-criticality )

\[ Q^2(z) \simeq Q_s^2 + \rho_{\text{dust}}^0 (1 + z) , \quad Q_s^2 > 0 \]

Matter (including exotic scaling dark matter)

\[ \rho_{\text{matter}} \sim \rho_{\text{dust}}^0 (1 + z)^3 + \rho_{\text{exotic}}^0 (1 + z)^{2+} + \ldots , \quad \alpha = a_0/(1 + z) \]

**Dilaton Dark Energy** (\( \alpha = \) string loop correction parameter)

\[ \rho_\phi \sim H^2 + \frac{Q_s^2}{a^2} + \frac{\rho_{\text{dust}}^0 + \alpha/2}{a^3} + \frac{\beta}{2 a^4} + \ldots = \mathcal{O}(a^{-2}) + \mathcal{O}(a^{-3}) + \frac{\beta}{2} a^{-4} + \ldots \]

Combination appearing in Hubble \( H(z) \)

\[ \rho_\phi + \rho_M \simeq |\mathcal{O}(a^{-2})| + \left( \frac{4 \rho_{\text{dust}}^0 + \alpha}{2} a^{-3} - \left( \frac{|e^{2\phi} J_\phi|}{2} + \ldots \right) a^{-4} \right) \]

**Hubble parameter** (for fit)

\[ H(z) = H^0 \left( \Omega_3^0 (1 + z)^3 + \Omega_5^0 (1 + z)^{\delta} + \Omega_2^0 (1 + z)^2 \right)^{1/2} , \quad \delta \approx 4 \]

Fits with data can constrain Liouville Q-Cosmology Dilaton Potential!
Comparison between models

- Future searches for high-z SNe (SNAP) decisive for picking up best model
- Further analyses from CMB data, measurements of equations of state, etc needed...

<table>
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<th>Model</th>
<th>$\chi^2$</th>
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<td>Q-cosmology</td>
<td>237</td>
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LHC SUSY detection prospects
and
non-critical strings

Q-Cosmology modifications of Boltzmann Equation for relic abundances


Boltzmann equation for thermal relics modified by a source $\Gamma$ term:

\[
\frac{dn}{dt} + \frac{\dot{a}}{a} n = \Gamma(t)n + \int \frac{d^3 p}{E} C[f], \quad \Gamma(t) \equiv \dot{\Phi} - \frac{1}{2} \left( e^{-\Phi} g^{\mu\nu} \beta_{\mu\nu}^{\text{Grav}} + 2 e^{\Phi} \beta^{\Phi} \right).
\]

**Modified expression for relic abundance**

\[
\Omega_{\chi} h_0^2 = (\Omega_{\chi} h_0^2)_{\text{no-source}} \times \left( \frac{\tilde{g}_*}{g_*} \right)^{1/2} \exp \left( \int_{x_0}^{x_f} \frac{\Gamma H^{-1}}{x} dx \right)
\]

with \((\Omega_{\chi} h_0^2)_{\text{no-source}} = \frac{1.066 \times 10^9 \text{ GeV}^{-1}}{M_{\text{Planck}} \sqrt{g_*} J} J \equiv \int_{x_0}^{x_f} \langle \nu \sigma \rangle dx.
\)

**NB:** Notice presence of non-critical/dilaton prefactor \( R = \left( \frac{\tilde{g}_*}{g_*} \right)^{1/2} \exp \left( \int_{x_0}^{x_f} \frac{\Gamma H^{-1}}{x} dx \right) \)
Q-Cosmology modifications of Boltzmann Equation for relic abundances

**NB:** Modified (off-shell) dynamics (temporal components):

\[ \rho + \Delta \rho = \frac{\pi^2}{30} T^4 \tilde{g}_{eff} \]

\[ \rho = \frac{\pi^2}{30} T^4 \, g_{eff}(T) \]

**NB:** only the degrees of freedom involved in \( \rho \) are thermal, while \( \rho + \Delta \rho \) are involved in evolution

\[ H^2 = \frac{8\pi G_N}{3} (\rho + \Delta \rho), \text{ hence } \tilde{g}_{eff} = g_{eff} + \frac{30}{\pi^2} T^{-4} \Delta \rho. \]

**NB:** Freeze-out point \( x_f \) modification:

\[ [g_* \equiv g_{eff}(x_f), \quad \tilde{g}_* \equiv \tilde{g}_{eff}(x_f)] \]

\[ x_f^{-1} = \ln \left[ 0.03824 \, g_s \frac{M_{Planck}}{\sqrt{g_*}} \, m_{\tilde{\chi}} \, x_f^{1/2} \langle \nu \sigma \rangle_f \right] + \frac{1}{2} \ln \left( \frac{g_*}{\tilde{g}_*} \right) + \int_{x_f}^{x_{in}} \frac{\Gamma H^{-1}}{x} \, dx . \]
Physical Consequences of Source Terms $\Gamma$

(a) if $\Gamma < 0$ at all times $\implies$ reduction of the relic density with time: predictions for supersymmetric models may be drastically altered, since parameter space is enlarged, $\implies$ more room for supersymmetry, probably beyond the reach of LHC (even in case of constrained minimal SUSY standard models with compact parameter spaces of embedding minimal SUGRA)

(b) if $\Gamma > 0$ at all times, relic density increases. Opposite situation, parameter space is shrunk, predictions can be very restrictive, to almost excluding supersymmetry, (especially if prefactor large number.)

In present era, matter and non-critical (off-shell) terms (c.f. numerical analysis of Q-cosmologies) are Important $\implies$ further numerical studies of relic densities required for studying SUSY detection prospects at LHC in non-critical string cosmologies.
The energy density carried by the dilaton and the non-critical terms (dashed-dotted red line), the deceleration (dashed blue line), the Hubble expansion rate (dashed-double dotted grey line) and the derivative of the cosmic scale factor (solid green line) as functions of the redshift in the range $0 < z < 1.6$ are displayed. Their values refer to the left $y$-axis. The ratio $|q|/g_s^2$ is also displayed (dotted brown line) with values on the right vertical axis.
Q-Cosmology Predictions on Cosmological Parameters

Left panel: The ratio $|\dot{q}|/g_s^2$ ($q$=deceleration, $g_s = e^\Phi$=string coupling) as function of the redshift for $z$ ranging from $z = 0.2$ to future values $z = -0.6$. The rapid change near $z \approx 0.16$ signals the passage from deceleration to the acceleration period. Right panel: The values of the string coupling constant plotted versus redshift value in the range $z = 0.0 - 1.0$.

**NB:** The string coupling drives the acceleration of the Universe asymptotically.
**Left:** In the thin green (grey) stripe the neutralino relic density is within the WMAP3 limits $0.0950 < \Omega_{CDM} h^2 < 0.1117$, for values $A_0 = 0$ and $\tan \beta = 10$, according to the **conventional calculation**. The dashed double dotted line (in blue) delineates the boundary along which the Higgs mass is equal to 114.0 GeV. The dashed lines (in red) are the $1\sigma$ boundaries for the allowed region by the $g - 2$ muon's data as shown in the figure. The dotted lines (in red) delineate the same boundaries at the $2\sigma$'s level. In the hatched region $0.0950 > \Omega_{CDM} h^2$, while in the dark (red) region at the bottom the LSP is a stau. **Right:** The same as in left panel, but according to the **non-critical-string calculation** in which the relic density is reduced.
Left: In the very thin green (grey) stripe the neutralino relic density is within the WMAP3 limits $0.0950 < \Omega_{CDM} h^2 < 0.1117$, for values of $A_0 = 0$ and $\tan\beta = 40$ shown in the figure, according to the conventional calculation. The thin dark (purple) region lying above is the same region according to the non-critical-string calculation with the reduction factor for the MSSM inputs shown in the figure. The remaining Higgs and $g - 2$ boundaries are as in figure 145. The hatched dark (cyan) region on the left is excluded by $b \to s\gamma$ data. Right: The same as in left panel, for $A_0 = 0$ and $\tan\beta = 55$. 
co-annihilation region

\(m_{1/2}=360\), \(m_0=215\), \(\tan \beta=40\), \(m_{\text{top}}=175\)
(Arnowitt-Dutta Ref. Point)

Supercritical String

\(m_{1/2}=400\), \(m_0=471\), \(\tan \beta=40\), \(m_{\text{top}}=172.7\)
(Nanopoulos Ref. Point)

[See. D. Toback’s talk on May 17]
$m_{1/2} = 400, \ m_0 = 471, \ \tan\beta = 40, \ m_{\text{top}} = 172.7$  

(Nanopoulos Ref. Point)

Key to distinguish from signals in co-annihilation region

$\tilde{u}_L \rightarrow u\tilde{\chi}_2^0 \rightarrow uZ\tilde{\chi}_1^0$

Generator-level Study

Further study with a detector simulation is under way.

[A. Krislock, A. Gurrola, B. Dutta, T. Kamon, D. Nanopoulos]
CONCLUSIONS & OUTLOOK

LHC and future (linear) colliders could shed light on the outstanding issues of the nature of the Cosmological Dark Sector (especially Dark Matter), but this is a highly model-dependent statement. **Nothing is certain, and careful interpretations of possible results are essential.**

But future looks promising...

**Future Theoretical Avenues:** Find correct SUGRA/string,brane model to constrain SUSY searches exploiting Dark Energy component of Universe.

**Future Experimental Avenues:** Definitely Particle Physics and Astrophysics will proceed together for the exciting years to come and provide useful and complementary physics input to one another: LHC, new terrestrial (linear) colliders, novel precision astrophysical (terrestrial & extraterrestrial) experiments (Auger, Planck, high-energy neutrino astrophysics etc.).